Frequency dependence of the speed of sound in air

G. P. Howell
Rolls-Royce plc, P.O. Box 31, Derby DE2 8BJ, England

C. L. Morfey
Institute of Sound and Vibration Research, University of Southampton, Southampton SO9 5NH, England

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Recent papers demonstrate some confusion over the frequency dependence of the speed of sound in air. The aim here is to correct the misunderstandings that have arisen.

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INTRODUCTION

References 1 and 2 are among recent papers providing accurate means of calculating the zero-frequency sound speed in air under varying atmospheric conditions. However, the authors have misinterpreted the principal conclusion of Morfey and Howell,[3] who went on to predict the frequency dependence of sound speed. The purposes of this letter are to explain and to clarify some aspects of the frequency dependence, and hence to correct the misunderstandings that have arisen.

I. CALCULATION OF SPEED OF SOUND

Morfey and Howell's calculation consisted of two parts. First, they found the zero-frequency sound speed c_0 for the atmospheric conditions of interest. Then they took into account oxygen and nitrogen vibrational relaxation effects, and hence obtained the speed of sound c_ϕ at the required frequency. This second step assumed that the effect of the relaxation processes could be added in the following manner:

$$\frac{1}{c_0}-\frac{1}{c_\phi}=\sum_r \frac{\alpha_r}{2\pi f_r},$$

where α_r and f_r are the attenuation coefficient and relaxation frequency, respectively, for each process. The latter were calculated by use of the model of atmospheric air in Ref. 4.

Figure 1 shows predictions obtained in this way[3] at 292 and 503 Hz, plotted in the form $[c_\phi/c_\phi(\text{dry})-1]$. This shows how the sound speed $c_\phi(f)$ varies with humidity, at each frequency. For comparison, the zero-frequency line is also shown.

II. EXPLANATION OF RESULTS

To understand the general trend of these curves, consider first the oxygen relaxation process alone. Figure 2 shows the variation of sound speed with frequency (at 20 °C) for 0% and 3% relative humidities. The sound speed at frequencies of order 200–500 Hz is lower at 3% relative humidity than it is at 0%, and this explains the initial dip in the finite-frequency curves in Fig. 1. As the relative humidity increases further, the quantity $[c_\phi/c_\phi(\text{dry})-1]$ begins to rise again.

When the nitrogen relaxation process is also present, a similar effect occurs as the associated relaxation frequency passes through the frequencies considered (at about 35% relative humidity for 292 Hz). In this case, however, the effect is smaller and more gradual, and it does not cause a dip in the $[c_\phi/c_\phi(\text{dry})-1]$ curve.

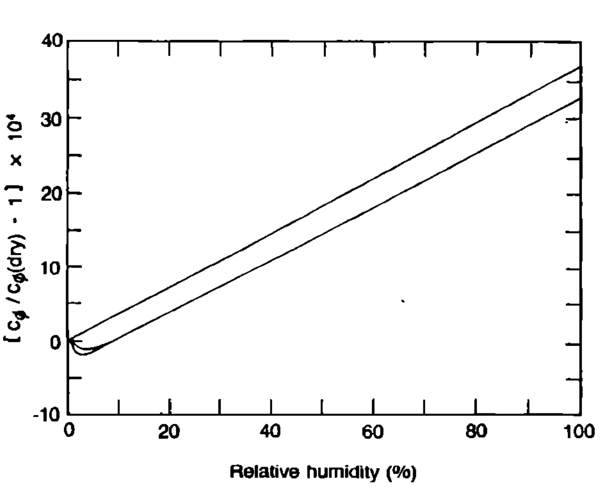

FIG. 1. The influence of humidity on sound speed in air at 20 °C, 1 atm. The upper curve is the zero-frequency prediction. The lower two curves, which are indistinguishable above 10% relative humidity, show predictions at 292 Hz (lower curve) and 503 Hz (upper curve).

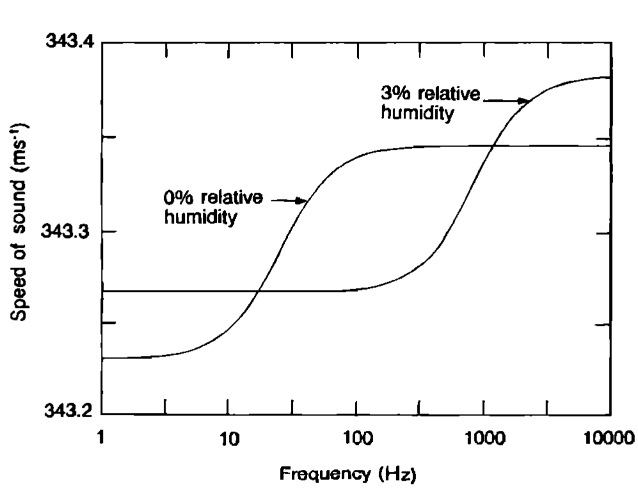

FIG. 2. Variation of sound speed with frequency at 20 °C, 1 atm, assuming oxygen relaxation only. Curves for 0% and 3% relative humidity are shown, but a similar trend continues as the relative humidity is increased further.

III. CORRECTION OF MISUNDERSTANDINGS

In Ref. 1, Wong and Embleton predict the zero-frequency curve $c_0/c_0(\text{dry})$. By omitting the " $-$ 1," they are able to express errors in parts per million (ppm). They note that their prediction differs from the lower curves in Fig. 1 by approximately 400 ppm for relative humidities above 10%.

This difference is wrongly attributed to a change in the dry-air specific heat ratio from 1.4007 to a new value of 1.3996. Indeed, by coincidence, $(1.4007/1.3996)^{1/2}$ does give a change of nearly 400 ppm. However, the correction cannot be applied in this way, since both dry and humid air are affected by a change in dry-air specific heat ratio.

The reason for the difference is that the lower curves in Fig. 1 are at finite frequencies (292 and 503 Hz). Wong and Embleton should instead compare their prediction with the zero-frequency curve in Fig. 1, and a close correspondence would then be found. The correction in specific heat ratio referred to above causes only very small changes in the ratio $c_\phi/c_\phi(\text{dry})$ (reaching a maximum of 12 ppm at 100% relative humidity).

Reference 2 shows a similar misunderstanding in its assertion that Morfey and Howell's method is applicable "only to sound-speed variation with humidity at 20 °C." The method is in fact capable of predicting sound-speed variations with all four variables—humidity, frequency, temperature, and pressure.

IV. CONCLUSIONS

Morfey and Howell's[3] method for calculating the speed of sound consists first in finding the zero-frequency sound speed, and second in adjusting this value to allow for vibrational relaxation effects. References 1 and 2 offer revised numerical constants for improving the accuracy of the first part of the method. A confusion between the two parts of the calculation has been corrected. The frequency-dependent part is itself open to improvements in accuracy, as more reliable measurements of attenuation coefficient become available.[5]

[1] G. S. K. Wong and T. F. W. Embleton, "Variation of the speed of sound in air with humidity and temperature," J. Acoust. Soc. Am. 77, 1710–1712 (1985).

[2] G. S. K. Wong, "Characteristic impedance of humid air," J. Acoust. Soc. Am. 80, 1203–1204 (1986).

[3] C. L. Morfey and G. P. Howell, "Speed of sound in air as a function of frequency and humidity," J. Acoust. Soc. Am. 68, 1525–1527 (1980).

[4] ANSI S1.26-1978, "American national standard method for the calculation of the absorption of sound by the atmosphere" (Acoustical Society of America, New York, 1978).

[5] A. J. Zuckerwar and R. W. Meredith, "Low-frequency absorption of sound in air," J. Acoust. Soc. Am. 78, 946–955 (1985).

Response to "Frequency dependence of the speed of sound in air" [J. Acoust. Soc. Am. 82, 375–376 (1987)]

George S. K. Wong

Division of Physics, National Research Council of Canada, Ottawa, Ontario K1A 0R6, Canada

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This response addresses the comments of Howell and Morfey on the misinterpretation by this author of their prediction of the effects of humidity on the velocity of sound in air [J. Acoust. Soc. Am. 68, 1525–1527 (1980)]. The misunderstanding has arisen due to the fact that the above authors superimposed their theoretical prediction curves, which have an ordinate unit of $(c_{\phi a}/c_\phi - 1)\times 10^4$, onto Harris' experimental data [J. Acoust. Soc. Am. 49, 890–893 (1971)], which has an ordinate unit of c_h/c_0, and with no information given on their zero-frequency curve [J. Acoust. Soc. Am. 82, 375–376 (1987)]. The misinterpretation is clarified, and this author now concludes that the theoretical prediction given by Wong and Embleton [J. Acoust. Soc. Am. 77, 1710–1712 (1985)] as well as the theoretical predictions of Howell and Morfey are in agreement to within 50 ppm.

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LIST OF SYMBOLS

c_ϕ	sound speed at any given frequency
$c_{\phi a}$	sound speed at any frequency, where a denotes dry air
γ_a	specific heat ratio of dry air
γ_w	specific heat ratio of water vapor, where w denotes water vapor
M_a, M_w	molecular weight of air and water vapor, respectively
	(The above quantities, as defined in Ref. 1, are shown here for reference)
c_0, c_h	speed of sound in dry and humid air, respectively
h	relative humidity, dimensionless